A PHOTOELASTIC STUDY OF THE STRESS DISTRIBUTION IN STIFFENED PLATING

Rockwell Holman

A PHOTO-ELASTIC STUDY OF THE STRESS

DISTRIBUTION IN STIFFENED PLATING

by

ROCKWELL HOLMAN

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Cambridge, Massachusetts
May 23, 1955

Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

In accordance with the requirements for the Degree of Naval Engineer, I herewith submit a thesis entitled: "A Photo-Elastic Study of the Stress Distribution in Stiffened Plates".

Respectfully yours,

Rockwell Holman
Lieutenant, junior grade
United States Navy

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Submitted to the Department of Naval Architecture and Marine Engineering on 23 May 1955 in partial fulfillment of the requirements for the degree of Naval Engineer.

ABSTRACT

The object of this thesis is to show the effect of a stiffener on the stress distribution in a flat plate, clamped at each end and centrally loaded with a single concentrated load. Models of three aspect ratios were constructed of Plexiglas and these models were loaded in a specially constructed load frame to produce the isoclinic pattern. The effect of the stiffener was found by comparing the pattern of the stiffened plate with that of the unstiffened plate.

The results of the study are generally inclusive, due to distortion in the optical system of the polariscope. However, there are trends indicated in the position of the isotropic points and in the slope of the isoclinics at the boundary, as well as the rate of change of direction of the principal stresses at the boundary that justify further study in the problem.

Thesis Supervisor: J. Harvey Evans

Title: Associate Professor of Naval Architecture

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TABLE OF CONTENTS

I Introduction.................................. 1

II Procedure.................................. 5

III Results.................................. 6

IV Discussion.................................. 7

V Conclusions.................................. 12

VI Recommendations.................................. 13

VII Appendix.................................. 14

A. Supplementary Introduction.............. 15

B. Details of Procedure.................. 20

C. Design and Construction of the Load Frame.............................. 24

D. Original Data.......................... 35

E. Literature Citations.................. 47

INTRODUCTION

This thesis is an attempt to show, through photoelastic phenomenon, the effect of stiffeners on the stress distribution in a flat plate under a single concentrated load. The complete stress distribution may be determined photoelastically from the isoclinics and the isochromatics. In a qualitative sense, the effect of a stiffener may be seen by study of the isoclinics alone through comparison with the isoclinic pattern in an unstiffened plate. Since the largest portion of the time devoted to this thesis was utilized in the design and construction of a loading frame and since certain limitations were found to exist in the particular polariscope used for the analysis, it has been found necessary to limit the extent of the study to determination of the isoclinic pattern for three aspect ratios. A description of the loading frame is to be found in Appendix II and the description of the modified polariscope is given under the Details of Procedure section of Appendix I.

The role of the stiffeners, in their ability to extend the range of stability of a flat plate, is well known and is susceptible to mathematical analysis. However, relatively little is known about the effect of stiffeners at less than critical load. This situation applies particularly to ship bulkheads, both longitudinal and transverse. It is a current practice to design these bulkheads on the basis of a normal water pressure loading and to treat deck loads, which act in the plane of the bulkhead, as secondary loads. Except for tank bulkh ads, the designed water pressure loading of the bulkhead is met only in time of damage to the ship and it is the deck loads which are the working loads. The Society of Naval Architects and Marine Engineers has requested information concerning the action of the stiffeners in distributing the deck loads into the sides and bottom of the vessel. The problem has been

approached by utilizing rectangular flat plates of various aspect ratios. Boundary conditions for the full scale bulkhead are extremely difficult to determine and for that reason the models used in this analysis have used the limiting condition of full clamping at the ends.

Previous Work.

In general, interest in the field of stiffened plating has centered around the problem of failure by instability. Considerable work, both experimental and analytical, has been reported in the literature. It is only natural that now that this type of failure is understood there should be interest arising in the behavior of the panels at loads below critical. Clearly, the interaction of plate and stiffener is a factor of significance at all loads.

The stress distribution in deep beams has been of increasing interest to civil engineers and work in this field is felt to have applicability to the field of naval architecture. Bulkheads in particular, function as deep beams under the action of deck loads. Reference (13) presents the analytical solution to the stress distribution in deep beams. Simple support is assumed and several types of loa ding are used. It is shown that the shear distribution at the quarter points is parabolic, as predicted by the Saint Venant theory, for aspect ratios of 2:1 or greater. (Aspect ratio is defined as the ratio of span to depth). The departure from this distribution at smaller aspect ratios is radical.

Photo-elasticity has been a favorite tool for instructional purposes in many texts of strength of materials and elasticity. It is particularly effective for beams and has been frequently used for the verification of the stress distribution predicted by the Bernoulli-Euler theory of flexure and to demonstrate the shift in neutral axis. For example, Professor Filon, in (16)

shows that, in a simply supported beam, loaded by a single central concentrated load and a pure couple, the isoclinic pattern in the vicinity of the point of zero bending moment should be a set of hyperbolas. The origin of the hyperbolas is on the neutral axis, at the point of zero bending moment. They intersect the free boundary at two symmetrical points, at the point of zero bending moment. Photo-elastic confirmation of this is also presented. The aspect ratio of the beam used for the experimental work was 12:1, which is considerably greater than the ratios used for this study. However, the beam loading approximates the conditions of the clamped end support, with the notable exception that warping of the cross sections is not restrained. Restraint of warping causes considerable aberration in the stress pattern predicted from the Bernoulli-Euler theory of flexure.

In the same paper Professor Filon investigates the area under the concentrated load for confirmation of the distribution of stress which he predicted in Philosophical Transactions, Vol 201, 1903. The effect of the concentrated load represents another departure from the Bernoulli-Euler distribution. Reference (3) contains a good discussion of the effect and the analytical methods available to describe it. In general it is treated by considering the stress distribution in a semi-infinite plate due to a single concentrated load. This distribution in superimposed on the flexure stress predicted by the Bernoulli-Euler theory and additional force systems added to conform to the boundary conditions. This was first presented by Carus Wilson in Philosophical Magazine, 1891, and, with refinements by G. G. Stokes, represents a means of explaining several of the phenomena observed photo-elastically.

The analytical solution of the stress distribution in a short clamped beam has not been presented to the best of this writer's knowledge. The restraint of warping characteristic of this type of support, as well as the

discontinuous nature of the concentrated load, make solution of the differential equation tedious. Probably the method of finite differences would lead to results which could then be combined with the semi-infinite plate distribution to account for some of the characteristics of the isoclinic pattern as presented in this study.

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SECTION II

PROCEDURE

The original concept of this study involved the determination of the stress distribution in a stiffened flat plate of photo-elastic material. It is regretted that the design and construction of the loading frame absorbed so much time that the isoclinic pattern became the primary objective. Isoclinics are the locus of all points of equal inclination of the principal stress directions, measured with respect to some fixed reference axis.

The effect of the stiffener, as reflected in the isoclinic pattern, was determined for three Plexiglas models. All models were of twelve inch span, but of various aspect ratios. These were 5:1, 3:1, and 2:1. The models were clamped in the loading frame and positioned in the field of the polariscope. A concentrated load of 400 pounds was applied and the isoclinic pattern traced.

The isoclinics were first determined for the unstiffened models. A central stiffener was then cemented in place along the shorter axis of symmetry and a second series of isoclinics determined for the stiffened models. In all cases the stiffeners were 1/4" x 3/4" Plexiglas and were attached by use of a volatile solvent, the principal ingredient of which was ethylene dichloride.

The isoclinics were determined at ten degree increments from 0 to 90 degrees and at 45 degrees as well. A more detailed discussion of the procedure is to be found in Appendix I.

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The effect of the stiffener, as reflected in the isoclinic pattern, was determined for three *Plexiglas* models. All models were of twelve inch span, but of various aspect ratios. These were 2:1, 3:1, and 3:2. The models were clamped in the loading frame and positioned in the field of the polariscope. A concentrated load of 400 pounds was applied and the isoclinic pattern traced.

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SECTION III

RESULTS

(1) It appears that, for the configuration of plate and stiffener used for this study, the stiffener does not materially affect the direction of the principal stresses, except in the immediate vicinity of the stiffener. The effect at the boundary is uncertain, due to limitations in the data.

(2) For a deep rectangular beam, clamped at each end and subjected to a single concentrated coplanar load at mid-span, there exist two negative isotropic points along each free boundary, located at about the quarter points. The isotropic points located along the loaded edge are further from the axis of symmetry than those on the lower, unloaded edge. These points represent the points of zero stress, which according to simple flexure theory should be located at the quarter points.

(3) It cannot be stated definitely that the stiffener has any effect on the location of the isotropic points. However, the information available from this study indicates that the stiffener has more effect upon the lower isotropic points than on the upper points. The 3:1 and 2:1 aspect ratio isoclinic pattern shows that the lower isotropic point has been moved toward the axis of symmetry and that the movement is quite pronounced, in contrast to the movement of the upper point. The plot of stress trajectories for the 3:1 aspect ratio unstiffened plate (Figure X) show that significant changes in the nature of stresses along the restrained boundary may result from such a shift of the isotropic point.

(4) There appears to be some effect from the stiffener in the rate of change of direction of the principal stresses along the restrained boundary. There is no clear-cut trend as a function of aspect ratio, but in general, the effect is opposite in direction in the upper and lower quarters of the depth.

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(2) For a deep rectangular beam, simply supported at each end and subjected to a single concentrated coplanar load at mid-span, there exist two separate isotropic points along each free boundary, located at about the quarter points. The isotropic points located along the loaded edge are further from the axis of symmetry than those on the lower, unloaded edge. These points represent the points of zero stress, which according to simple flexure theory should be located at the quarter points.

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IV. DISCUSSION OF RESULTS

The validity of the results depends to a large extent upon the accuracy of the data. The accuracy of the data presented in this study is felt to be unknown within precise limits, but, in general, is probably adequate for evaluation of the effect of the stiffener, in a qualitative sense. Figure IX shows the distortion pattern of a one-inch network as seen in the viewing glass. The network was placed in the plane of the model. This type of distortion is known as "pin-cushion distortion" (9) and results from radial variation in magnification. Professor Sears, in (9) states that this can be corrected by proper location of the diaphragms, but, for this system, this was not possible. The situation was circumvented by sketching the isoclinic pattern at several points in the model by positioning the points at the center of the light circle. The composite pattern was determined by placing the various patterns in their proper relationship and then tracing the final pattern. Distortion was still present, however, and small corrections were made to make the isoclinics continuous. The results were not perfect, as seen in the exaggerated length of the 3:1 and 5:1 aspect ratio patterns.

A second source of inaccuracy is found in the clamping arrangements. This causes a three-dimensional stress system to exist in the vicinity of the boundary. The isoclinics were poorly defined in this region, especially at about mid-depth. This condition may have been due to a slow rate of change of the direction of the principal stresses. This latter condition probably is responsible for the lack of definition experienced with the 2:1 aspect ratio plate. In general, however, it is felt that

the isoclinics are correctly located within one-eighth of an inch and about five degrees at the boundaries and that probably better results are achieved in the interior of the plate.

The stiffener has very little effect in the location of the isoclinics and, to a large extent, this explained by the nature of the loading. The load was applied to the plate, in order to maintain a two-dimensional stress system and under these conditions, the effect of the stiffener would be limited to the region of plastic flow. If, however, two stiffeners had been used, on opposite sides of the plate, the stiffeners could have borne part of the load without distorting the two-dimensional stress system and under this condition, the effect of the stiffener would have been greater.

The position of the isotropic points may be predicted, at least qualitatively, in the sense that the relative position of the points on the upper and lower boundaries at each quarter point may be rationalized. At first thought, "intuition" might lead one to expect that the upper point would move toward the axis of symmetry, due to rotation of the cross section under the effect of pure bending. Shear deflection would tend to increase the shift. However, the deflection of the 3:1 aspect ratio plate under the 400-pound load is only 0.072 inches and the rotation of the cross section at the point of zero bending moment is only 0.018 radians. This would indicate that the relative displacement of the upper and lower points, due to pure bending, is only 0.072 inches, which is much too small to be detected in the polariscope. Furthermore, the relative displacement due to deflection is opposite in direction to that observed. It is also true that displacement of the order observed would be reflected in the distortion of the network scribed on the surface of the model. No such distortion was observed.

The Bernoulli-Euler theory of flexure, otherwise known as simple beam theory, is presented in texts of strength of materials and may be used to predict the point of zero bending moment. The equations of elasticity may also be used to predict the point of zero stress along the upper and lower boundary. These are coincident with the point of zero bending moment, of course. However, though certainly more elegant, the latter method requires the solution of a fourth degree homogeneous differential equation. Due to the discontinuous nature of the concentrated load, it is necessary to use a Fourier series or the method of finite differences to solve the equation. The latter method is used in (13), for example. In an attempt to arrive at an analytical solution to the stress distribution, this writer has utilized a stress function composed of the terms of second, third, and fourth order polynomials to describe the stress distribution in a cantilever loaded with a concentrated load and a moment at the free end. The resemblance of this case to the clamped beam is close, since the clamped end of the cantilever corresponds to the axis of symmetry of the beam and the applied moment corresponds to the clamping moment of the beam. However, this does not represent any increase of accuracy over the simple beam theory for the purposes of predicting the point of zero stress along the free boundaries. Both methods of attack place the isotropic point at the quarter point.

According to simple beam theory, the upper edge of a clamped prismatic beam centrally loaded with a concentrated load is subjected to a linear variation of stress. The stress is tensile at the boundary, decreases to zero at the quarter points and reaches maximum compression at the midpoint of the beam. The stress on the lower boundary is similar, although of opposite character. It is compressive at the boundary and reaches

maximum tension at the mid-point. The isoclinic patterns of Figures VI through VIII show that the isotropic points are not exactly at the quarter points and it remains to account for this.

Frocht, in (3) describes the approximate solution for the stress distribution in a simply supported beam as presented by Carus Wilson in Philosophical Magazine, 1891 and modified by G. G. Stokes. This solution is built up by the superposition of the semi-infinite plate stress pattern on the simple flexure stresses. This requires, for equilibrium, the presence of radial compressive boundary loads, which do not actually exist. Therefore, a third superposition of radial tensions is made. The principle of Saint Venant justifies the substitution of the statically equivalent force system. This is a vertical force of magnitude P and two oppositely directed horizontal forces of magnitude P/π . These are located at the point of application of the load.

The stress trajectories of the 3:1 aspect ratio plate, Figure X, show strong similarity to the co-axial and radial stress trajectories, which are characteristic of Flamant's solution to the case of a concentrated load on a semi-infinite plate. If we substitute the three-force statical equivalent of the concentrated load, we see that the two horizontal forces create compression in the upper fibers of the beam. Whereas in the simply supported beam this system results in a secondary bending moment, it seems more likely that for the clamped beam the force is carried directly into the boundary as compression. The existence of a compressive stress in the upper fiber of the beam, superimposed on the stress pattern of pure bending, results in the upper isotropic point moving toward the support. Such movement would explain the location of upper isotropic point in the isoclinic patterns of Figures VI through VIII.

The stress pattern of the clamped beam is only approximated by the assumption of shear support and applied moment at the boundaries. The loading frame, by the nature of its construction, creates a more or less uniform tension in the loaded beam due to restraint of translation of the ends of the bbeam and a more complex stress system due to restraint of warping. These stresses will, of course, affect the position of the isotropic points. The uniform tension tends to shift the upper point toward the axis of symmetry and the lower point in the opposite direction. The restraint of warping stresses are tensile in the upper section of the plate and compressive in the lower, hence the lower point tends to shift toward the supports and the upper point toward the axis of symmetry. The final balance of these stresses determines the relative position of the isotropic points. It is felt that the clamping area determines, to a large extent, the relative importance of the tension and restraint of warping stresses.

The stiffener is seen to affect the position of the isotropic points for all the aspect ratios, although the effect is most pronounced for the 5:1 aspect ratio. The upper point is shifted toward the axis of symmetry and the lower point in the opposite direction. The 3:1 aspect ratio shows that the shift is negligible for the upper point and that the lower point has moved toward the axis of symmetry. The 2:1 ratio shows the same trend, the lower point being shifted still further toward the axis of symmetry. This indicates that, effectively, a compressive stress is imposed by the stiffener on the lower edge of the plate and that its magnitude varies with depth of the plate.

V. CONCLUSIONS

1. The accuracy of the data precludes any quantitative analysis of the effect of a stiffener upon the stress distribution in a plate clamped at the ends and loaded with a single concentrated load at mid-span.

2. The addition of a stiffener, to such a loaded flat plate, if not in contact with the load has negligible effect on the direction of the principle stresses.

3. The stiffener may have some definite effect on the actual location of isotropic points, which theoretically exist on the free boundaries at the quarter points.

4. The photo-elastic method represents a convenient method for analysis of plates and deep beams.

VI. RECOMMENDATIONS

1. It is recommended that this photo-elastic study be continued and that emphasis be placed on:

   a. accurate determination of the isotropic points,

   b. determination of the isochromatics, leading to the complete solution of the stress distribution,

   c. determination of the rate of change of inclination of the principal axes at the boundary,

   d. utilization of other boundary conditions, with especial emphasis on an elastic foundation.

2. Future studies in this direction should be made with models of a more sensitive material, such as one of the Bakelite or Catalin plastics. Recent development of a suitable adhesive by the Armstrong Corporation makes the construction of built-up models feasible. Stiffeners should be located on both sides of models subject to co-planar loads.

3. It is recommended that the load frame be modified by substitution of a steel loading beam to reduce the deflection and by reduction of the size of the weighing tank to about half the present size.

SECTION VII

APPENDIX

APPENDIX A

SUPPLEMENTARY INTRODUCTION

Photo-elastic Concepts.

Photo-elasticity depends upon the characteristic ability of certain materials to exhibit temporary double refraction. These materials, of which glass, gelatin, Plexiglas and Celluloid are a few common examples, are optically anisotropic under stress and light travels through them along the planes of principal stress. In effect, then, they act as a polarizer and will resolve incident light into two mutually perpendicular planes.

Light travels through the photo-elastic model along the principal planes and is refracted according to the well-known Snells Law of refraction. The index of refraction along a principal plane is a function of the stress level on that plane, as well as the level on the perpendicular plane. Therefore, the velocity of light along each principal plane may be different. Except for the case of uniform tension or compression and pure shear, the velocity of light is a continuous function, varying from point to point along the stress trajectory. For this reason, light falling upon a stressed photo-elastic model is separated into two perpendicular components which travel through the model at different velocities and emerge out of phase.

The polariscope utilizes this characteristic for the determination of the isoclinics, which are the loci of points at which the inclination of the principal stress planes is a constant, and for the isochromatics. These are the loci of points at which the difference of the principal stresses is a constant. It follows, therefore, that they represent lines along which the maximum shear is a constant.

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A basic polariscope might consist of a light source, a polarizer for

creating polarized light and an analyser, which resolves the two components of light emerging from the model into a plane. A somewhat more refined installation would have a lens system for concentrating and collimating the light and two quarter-wave plates for producing circularly polarized light from a monochromatic source. Descriptions of various arrangements are available in any text on the subject. The polarizer and analyser used for this study were twelve-inch Polaroid sheets. There is no physical difference between a polarizer and an analyser; they are so named to describe their function in the polariscope.

Light leaving the polarizer may be represented as a vector, the magnitude of which may be written as:

$$V = a \cos (pt) \qquad (1)$$

where V = vector

a = amplitude of vibration

p = propagation factor = $2\pi f$

t = time, seconds

This light, upon entering the model, is broken into two components;

along the P axis; $a \cos(pt) \cos \theta$ (2)

along the Q axis; $a \cos(pt) \sin \theta$ (3)

where: θ = angle between the plane of the polarizer and the P axis

P = principal stress of largest algebraic value

Q = principal stress of least algebraic value

The light emerging from the model is characterized by a phase difference existing between the two components. This may be represented by a time displacement, t_1 for the P axis and t_2 for the Q axis. The components are then:

$$\text{along the P axis; } a \cos \theta \cos p(t-t_1) \tag{4}$$

$$\text{along the Q axis; } a \sin \theta \cos p(t-t_2) \tag{5}$$

The analyser may be aligned with its plane of polarization perpendicular to that of the polarizer or parallel to it. For the former arrangement, the two components above are combined by the addition of their components in the direction of the analyser axis. These components will be of the opposite sense and are as follows:

$$\text{P axis; } a \cos \theta \sin \theta \cos p(t-t_1) \tag{6}$$

$$\text{Q axis; } -a \cos \theta \sin \theta \cos p(t-t_2) \tag{7}$$

These two components are added in the analyser and the following expression may be easily developed by the use of trigonometric identities:

$$V = a \sin 2\theta \sin \frac{p(t_1-t_2)}{2} \quad \sin \frac{p(t_1-t_2)}{2} \tag{8}$$

The intensity of light is proportional to the square of the amplitude and therefore we see that there are two possibilities under which extinction may take place. If $\theta = 0$, which requires that the plane of polarisation coincide with the direction of one of the principal stresses, then extinction will occur. Recognizing that the quantity $\sin \frac{p(t_1-t_2)}{2}$ is proportional to the phase shift, we see that if the phase, timewise, is equal to the period of the light, extinction will also occur. This follows from the definition of p, the propagation factor.

The first of these criteria is the basis for the existence of isoclinics. The second criteria is the basis for the formation of the isochromatics, and depends on the difference of the principal stresses, the thickness of the model and the wave length of the incident light.

This study conserns itself with the isoclinics and therefore some further discussion of these lines is justified. If white light is used, the isoclinics are visible as a black band, more or less sharply defined depending on such

things as the rate of change of the direction of the principal stresses, and improper loading; e.g., a three-dimensional stress system. For a relatively insensitive material, such as Plexiglas or glass, the isochromatics will not appear to any extent at the low loads required for isoclinics. Since the actual magnitude of the stress is not a factor in formation of isoclinics, an infinitesimal load is theoretically sufficient. However, in practice, light loads are required to overcome initial or residual stress. For sensitive materials, some workers favor a relatively heavy load. (2)* The formation of any particular isoclinic is independent of load and isoclinics of any parameter are seen by rotating the polarizer and analyser together to the desired angle.

There are some properties of isoclinics which should be brought out.

1. Isoclinic lines do not intersect, except at an isotropic point. At such a point, the principal stresses are equal in magnitude. If the magnitude be zero, the point is further classified as a singular isotropic point. If, at an isotropic point, the parameters increase clockwise, the point is described as a negative isotropic point and if the direction of increasing parameter is counter clockwise, then it is positive. There are several other categories, for which see (16).

2. The parameter of an isoclinic which intersects a free boundary at other than an isotropic point is defined by the inclination of the normal or tangent to the boundary at the point of intersection.

3. Mesnager's theorem states that the principal stresses tangent to a given stress trajectory are a maximum or minimum where an isoclinic intersects the trajectory at right angles. It follows that, at a free boundary, where ever

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* Numbers in parentheses refer to references in the Literature Citation in Appendix I, (F).

the isoclinic intersects it normally, maximum or minimum stress exists.

4. Isoclinics may be used to determine the stress trajectories.

B. DETAILS OF PROCEDURE

Description of Apparatus

The load frame, which absorbed the greater part of the time devoted to this thesis, is described in Appendix II.

The polariscope, as modified, consisted of a light source, which utilized a 500-watt incandescent projection lamp, a 4-inch condensing lens, an 8-inch collimating lens and a 12-inch diameter sheet of Polaroid, mounted in a calibrated rotating ring. A water bath was located between the light source and the condenser lens and a diaphram was located at the center of least confusion of the condenser lens. The collimated light passed through the model and a second Polaroid sheet, was then collected by a 8-inch condensing lens and focused on a ground glass screen. Tracing paper was placed on the screen and the isoclinics traced thereon.

The modifications to the polariscope are largely in the two large sheets of Polaroid which served as polariser and analyser respectively in the order described above. They replaced two Nicol prisms which were located at the centers of least confusion of the two condensing lens. The Nicol prisms, in themselves were entirely satisfactory. It was found, however, that the first of the 8-inch lenses was carrying a residual or frozen stress which created its own isoclinic pattern and distorted the pattern in the plate. By dismantling the laboratory's Polaroid polariscope, used for demonstrations, and placing the Polaroid sheets between the two 8-inch lenses, the effect of the frozen stress was removed from the isoclinic pattern.

The use of the Polaroid sheets created one additional step in the work required. The polariser and analyser had to be aligned so that the plane of polarization of the polarizer was either vertical or horizontal

when the scale reading was zero degrees. Fortunately, one of the sheets was loose in its frame and could not be tightened. This was used as the polarizer. A diametrally loaded Bakelite ring was placed in the field with the axis of loading vertical. The Polaroid polarizer retaining frame was set at zero degrees on the scale scribed against an index line and the loose sheet rotated with the analyser to bring out the vertical isoclinic in the ring. The polarizer sheet was then fixed in position with respect to the retaining frame by several strips of cellophane tape.

Procedure

The determination of the isoclinics required that the polarizer and analyser transmitting axes be mutually perpendicular and that this relationship be maintained as the polarizer is rotated to produce isoclinics of different parameters. This is the arrangement which produces maximum extinction if no model is in place. The polarizer, since it had been set at zero degrees for the vertical direction and was marked in five degree increments to 90 degrees, was felt to be adequate for determining the parameter of the isoclinics. Therefore, to produce any isoclinic, as for example the 30 degree isoclinic, the polarizer was rotated to 30 degrees and the analyser was in turn rotated to produce maximum extinction. The amount of rotation was marked on the retaining ring of the analyser and over a series of runs, from 0 to 90 degrees, it was found that the analyser rotation, as judged by observing the point of maximum extinction, was reproducible within one-half of a degree for any angle and, in general, was better than that. Therefore, to reduce the time required to take data, the analyser setting was marked and used without further check for each angle.

The models were constructed of 1/4" Plexiglas, a photo-elastic material of low sensitivity. Three aspect ratios were used; namely 5:1, 3:1, and 2:1, based on a common span of 12 inches. The overall span was 14 3/8", there being 1 3/16" clamping area at each end. The stiffeners were, in all cases, 1/4" x 3/4" Plexiglas and these were attached to the model along the shorter axis of symmetry. The stiffeners were "welded" to the plate by soaking the contact edge in the solvent for about five minutes. The solvent was furnished by Forest Products, Inc. of Cambridge, Massachusetts and it is understood that the principal ingredient was ethylene dichloride. The stiffeners were placed in position on the model and pressed with weights to remove any bubbles on the interface. Before it was completely set, a few drops of the solvent were placed on the line of contact formed by the plate surface and the side of the stiffener. These were allowed to run evenly over the entire length and resulted in a fillet as well as removing the effect of any irregularities in the edge of the stiffener.

Each model was marked with a one-inch grid and then placed in the load frame. The models were clamped between two pieces of hot-rolled steel which in turn were clamped by the inner posts of the load frame. It is felt that the clamping bolts were set up uniformly at about 10 pound-feet torque and that the tension in the bolts was about 1500 pounds. The bolts were set up with a socket wrench which had a 10-pound weight attached at the end.

The square grid was used to determine the amount of distortion in the system and to aid in positioning the model in the field to assure complete coverage. Due to the distortion in the lens system, it was necessary to reposition the model at least four times to adequately cover

half the model. If it had been possible to correct the "pin cushion" distortion resulting from non-uniform magnification, the amount of time required would have been cut in half and the accuracy of the data increased considerably.

The load was applied to the model by a 1/2" diameter load point, acting on the plate through a 3/16" diameter load pin. The axis of the load pin was normal to the plane of the plate. The load pin and pointer were accurately aligned over the plate to insure two-dimensional stress distribution. For the same reason, the load did not bear on the stiffener, for the stiffened models. The weight of the weighing tank by itself was found to be sufficient to produce isoclinics in all the models. Further loading seemed to sharpen the isoclinics and for the sake of uniformity, a load of 400 pounds was used. Water was added to the weighing tank by operating the three-way valve, which was located at the strain indicator.

Using white light, the isoclinics were determined at ten degree increments from 0 to 90 degrees and at 45 degrees. The 90-degree isoclinic coincides with the 0-degree isoclinic and was used as a check point. Similarly, the 45-degree isoclinic is symmetrical about the axis of symmetry for a model loaded on the axis of symmetry and this isoclinic was used to check the symmetry of loading as well as the accuracy of the zero-setting of the polarizer.

C. DESIGN AND CONSTRUCTION OF THE LOAD FRAME

Considerations

There are several requirements for any loading system and these might be loosely sub-divided into those peculiar to the particular system and those which apply regardless of how the load system is to be used. The loading frame which was designed and constructed incident to this thesis was envisioned as an addition to the equipment of the Ships Structure Laboratory, Massachusetts Institute of Technology, and for this reason, greater attention to detail was given than would otherwise be the case for such a small thesis. The considerations which were entertained are listed below. Some of these requirements have not been met in the sense that modifications or additions not absolutely required for this particular thesis have not been made.

1. The load frame must be able to apply either tension or compression.

2. Application and removal of load must be smooth.

3. The applied load must be easily reproducible.

4. The calibration of the load frame should be simple and direct reading.

5. The loading and the model must be maintained in a position normal to the axis of the light.

6. Deflection of the load frame should be of at least one order of magnitude lower than that expected in the model in order that deflection of the model is not influenced sensibly by deflection of the frame and that the direction of loading does not change.

7. The frame must be able to adjust easily to different sizes and shapes of models.

8. Remote loading from the viewing position of the polariscope is highly desirable.

9. Horizontal and vertical movement of the frame is necessary to accurately position the model.

Description

Figure I is the original plan of the loading frame, designed to produce a concentrated load of 1500 pounds at the load pointer. The existing design differs in small details from the drawing; the most prominent of these is the increased height of the vertical posts. These have been increased in length to 30 inches. There is indicated on the plan a system for maintaining the upper and lower edges of the models in a plane. This scheme was abandoned from economic considerations since it was not entirely certain that it would be required.

The vertical posts of the load frame are 3" x 0.258" structural Aluminum channels. Calculations indicated that the pivot pin reaction would not exceed 1000 pounds and that the deflection of the outer posts would therefore not exceed 0.0053 inches. It was felt that it was necessary to remove the effect of this deflection from the model supports, especially since the deflection is not symmetrical. For this reason, the inner posts were added for the purpose of retaining and supporting the model. The 3/4" holes in the outer posts are spaced 2 inches apart, the holes in one pair being 1 inch lower than the other pair. This permits adjustment of the height of the loading beam in 1-inch increments.

The steel pivot pin, about which the loading beam pivots, is supported by two brass bushings in the 3/4" holes mentioned above. The

bushings are a light press fit in the holes and were felt to be necessary to prevent elongation of the holes in the aluminum channels with resulting loss of accuracy. Deflection of the steel shaft is calculated at 0.002 inches, taking account of shear deflection and assuming simple support in the bushings.

The load beam is fitted with a Norma-Hoffman R312-LL roller bearing rated at 2910 pounds load at 25 revolutions per minute. This is obviously better performance than is required, but several considerations dictated its selection. It was desired to keep the spacing of the vertical posts at about 1/2 inch in order to reduce the shims required to clamp a 1/4-inch model. The load pointer would be centered in the post spacing and therefore the bearing should not exceed the thickness of the beam, which was limited by the post spacing. In addition, it was recognized that the loading beam would be the critical part of the load frame, due to this limitation on its thickness, and therefore the diameter of the bearing should be as small as possible. Ball bearings, which would be preferable from the low-friction consideration, are not available under these limitations.

The maximum moment anticipated in the loading beam is 16,100 pound-inches and the maximum flexure stress is 31,000 pound per square inch. The beam is 24S-T4 alloy and the yield strength of this material is 48,000 pounds per square inch. Ultimate strength is 68,000 pounds per square inch. These figures indicate a rather high stress level and investigation of the deflection to be expected shows that it will be a bout 1.00 inches at the free end. However, investigation and experience has shown that loads of over 1000 pounds are seldom required and that

in general, an upper limit of 500 pounds might be accepted. In service, therefore, the probable deflection will be about 0.30 inches.

The cross feed mechanism was made up by the Ace Machine and Tool Company of Cambridge, Massachusetts from sketches furnished by the writer and no formal drawings are available for presentation. Figure V, however, shows the roller bearings and hold-down mechanism as well as the lathe screw used to move the bed plate and loading frame across the field of the polariscope. Steel bearing strips are attached on the under side of the bed and the rollers bear directly on these. They were felt to be necessary since the bed plate is of "O" temper aluminum, the hardness of which is only about 28 Brinnell.

The requirement for smooth loading was met by adoption of a hydraulic loading system. This consisted of a 56-gallon tank which was suspended from the loading beam by a slider identical in form to the one used to load the model. The tank is open at the top and is fitted with two sill cocks at the bottom. Water is added or removed from the tank by a three-way valve and system of hoses connecting with the city water mains. The three-way valve is located at the viewing screen and may be seen in Figure II. In the "Load" position, water is admitted to the tank directly through the valve. In the "Unload" position, water is directed to an ejector, which in turn removes water from the tank at about 2.5 gallons per minute. The ejector is a commercial device used to drain home washing machines. It operates by a reduction in pressure resulting from flow through an orifice, and represents the least expensive and most trouble-free type of pump available.

Direct measurement of the load is achieved by use of the SR-4 Type A-7 electric strain gage. The load pointer is cylindrical except for two

small axial flats machined parallel to the keeper groove which holds the 3/16-inch load pin in place. One strain gage is mounted on each of these and placed in opposite arms on an external bridge circuit. Two more gages of the A-7 type are located in the other arms of the bridge and are used for temperature compensation. Figure IV shows the slider, load pointer and load pin in position. The compensating gages are mounted on the flat bar attached at the upper corner of the slider.

The maximum strain in the pointer is 745 micro-inches per inch, under 1500 pounds load. Calibration of the axial gages at their rated gage factor of 1.95 led to constants of 0.49 micro-inches per inch per pound and 0.496 micro-inches per inch per pound respectively for the two gages separately. By operating at a gage factor of 1.92, with the axial gages in opposite arms of the bridge, a calibration of 1.0 micro-inches per inch per pound was attained, leading to extremely simple determination of load magnitude. The external bridge was connected to a Baldwin Type L Strain Indicator and it is estimated that the load is known to within 2.5 pounds over the range, based on the resolution of the strain indicator.

Results

The loading frame has been used only for compressive loads and in this respect it has performed as expected. The method of clamping the model appears to be satisfactory and it is not anticipated that any difficulty will be met in adapting the frame for tensile loads, pure bending or any other type of loading which may, in future work, be required.

The loading beam and water tank have proved to be a satisfactory arrangement. Smooth variation of the load as well as reproducibility have been demonstrated. Deflection of the beam is felt to be excessive

and it is difficult to achieve zero load, due to the weight of the tank and beam. The load pointer has been very satisfactory and represents an improvement over several loading frames described in the literature in that calibration of the tank is not necessary. In addition, it is not necessary to determine to load as a function of the lever arm of the tank.

Recommendations

Probably the most significant change which might be made in the load frame is in the load beam and water tank, in order to facilitate achieving zero load and reduce the deflection of the beam. The size of the tank could be reduced to thirty gallons or less and thus cut its weight nearly in half. The use of steel for the beam would reduce the deflection by about one third, but at the cost of some of the weight gained by reduction of the size of the tank. Counter balancing of the tank would then be practical and zero load could be achieved.

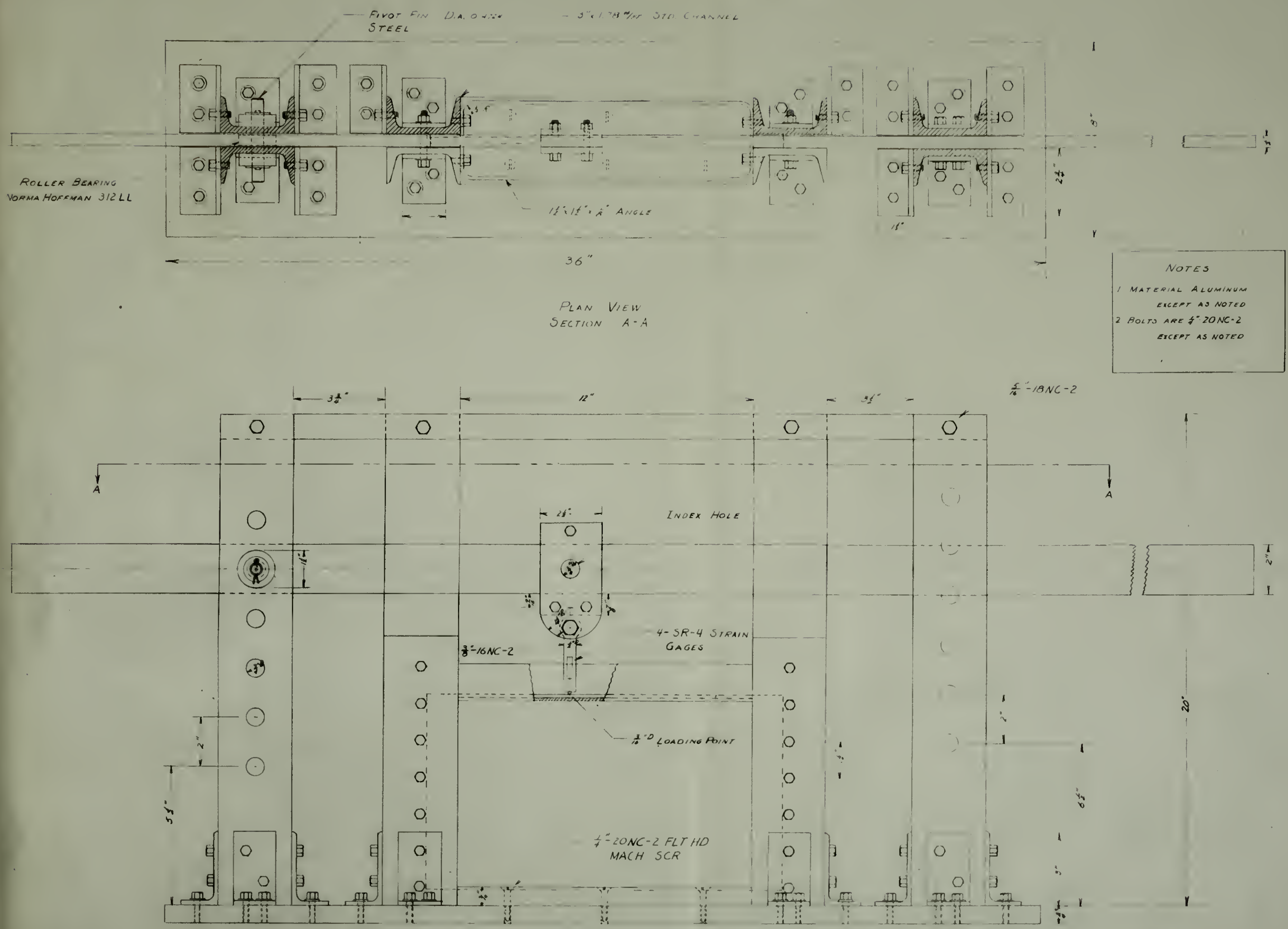


Figure I
Elevation View of Loading Jig

R.H.
2/10/55

FIGURE II

General View of the Polariscope and Load Frame

Note the three-way valve in the foreground, close to the viewing screen and the strain indicator.

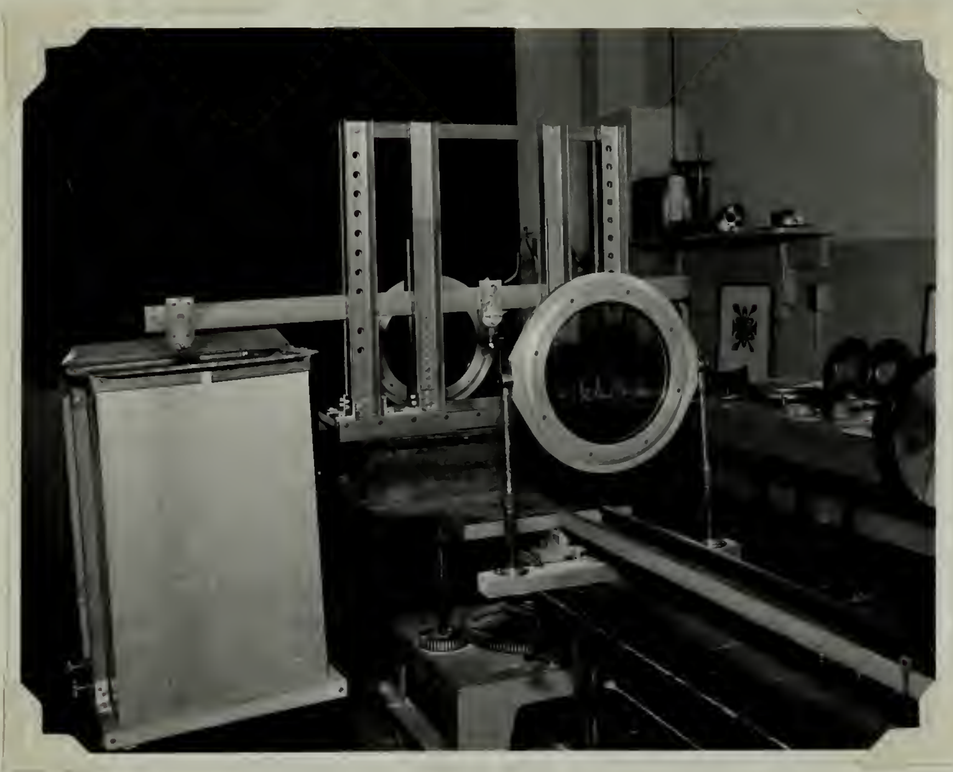


FIGURE III

Load Frame and Weighing Tank

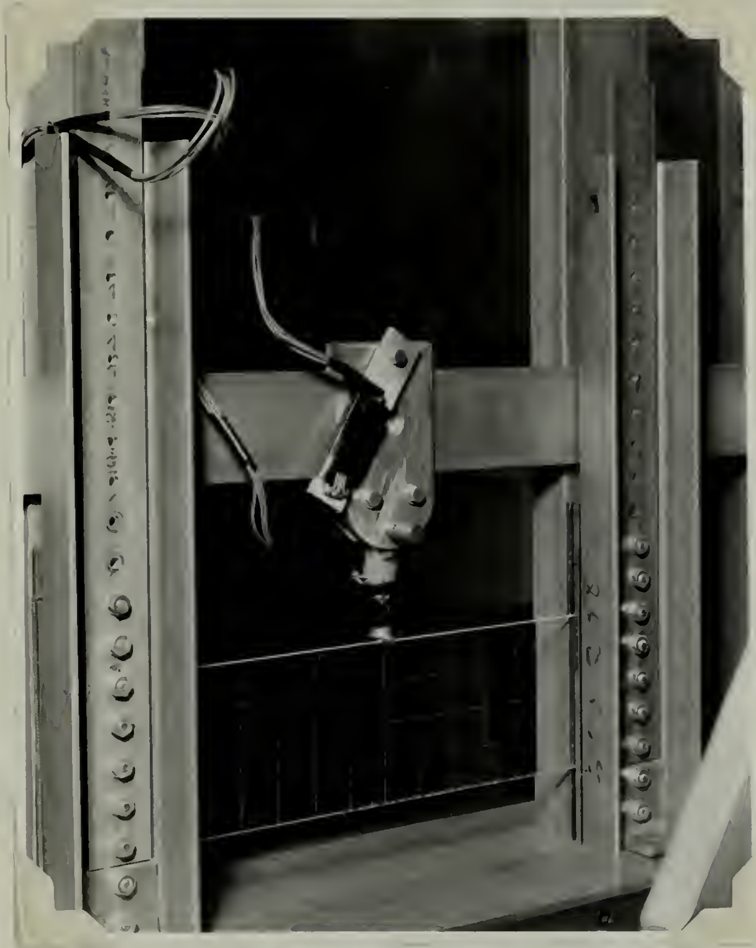


FIGURE IV

View of Load Pointer and Pin

Note the dummy compensating strain gage strip and the method of clamping the model.

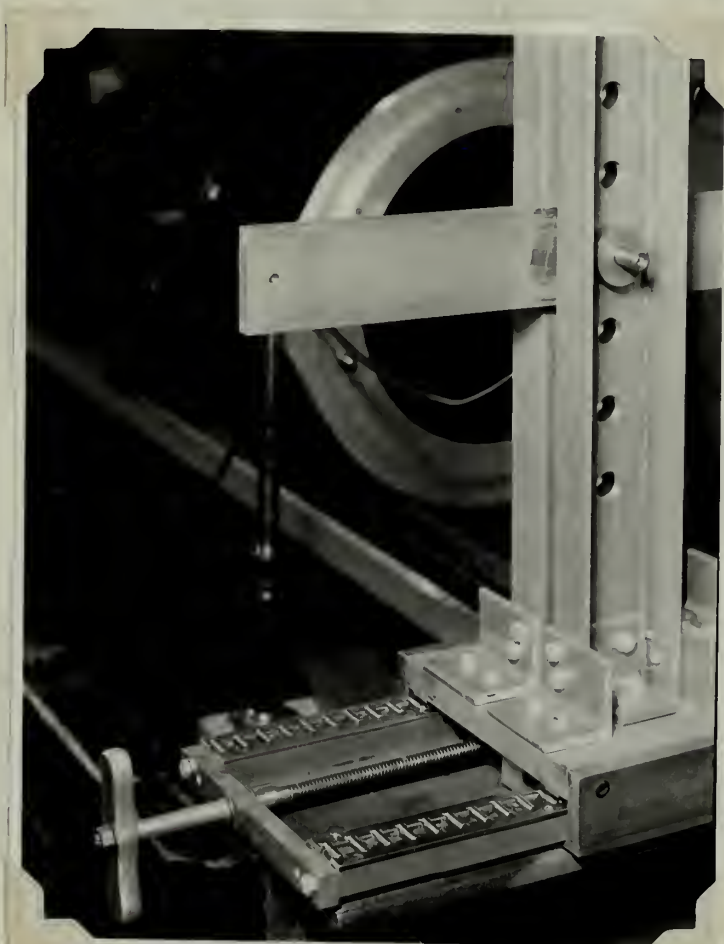


FIGURE V

View of Crossfeed Mechanism

Note the rollers and the hold-down clips, which keep the bed plate against the rollers under the moment of the applied load.

APPENDIX D.

ORIGINAL DATA

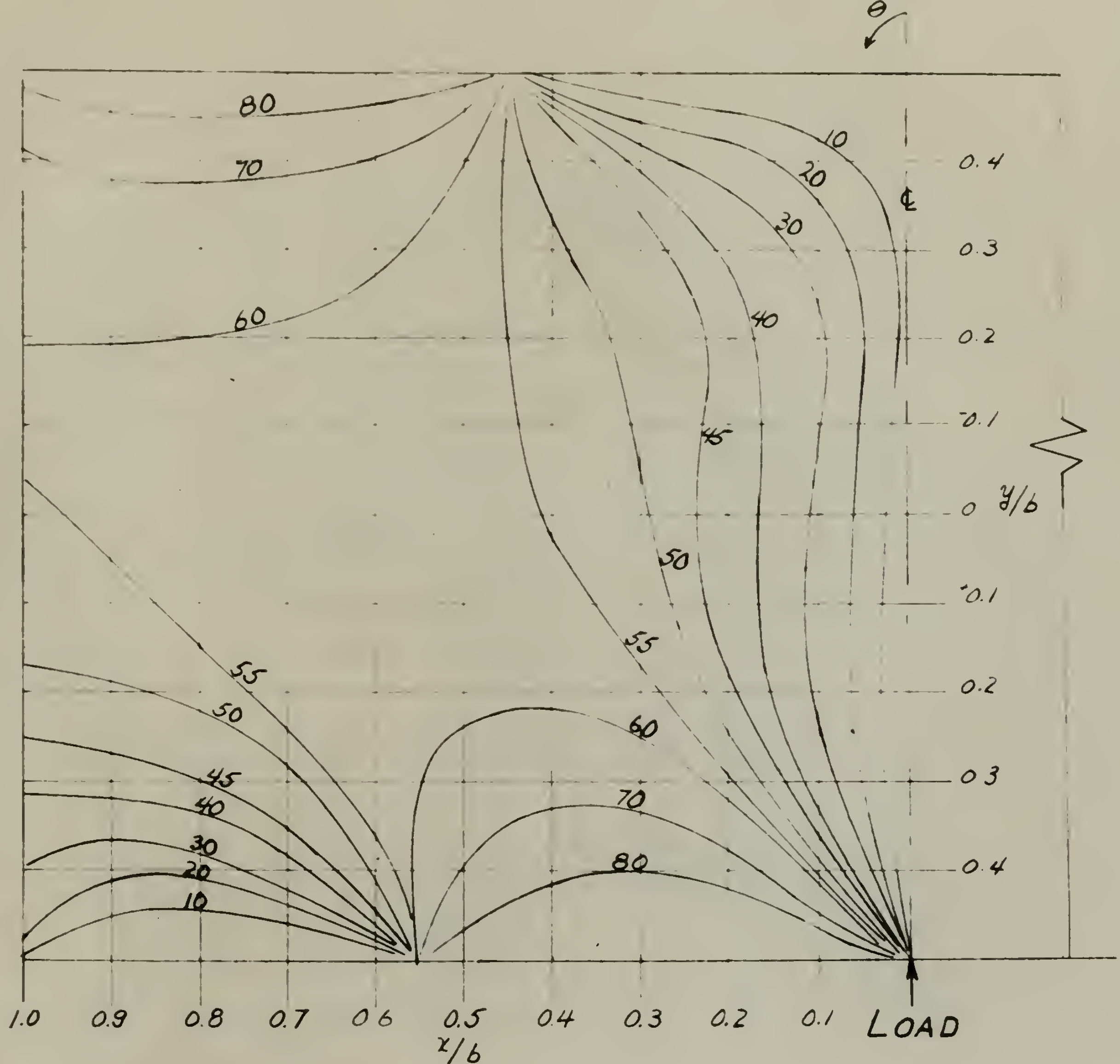


Figure VI (a)
ISOCLINIC PATTERN
Aspect Ratio 2:1
Unstiffened Plate

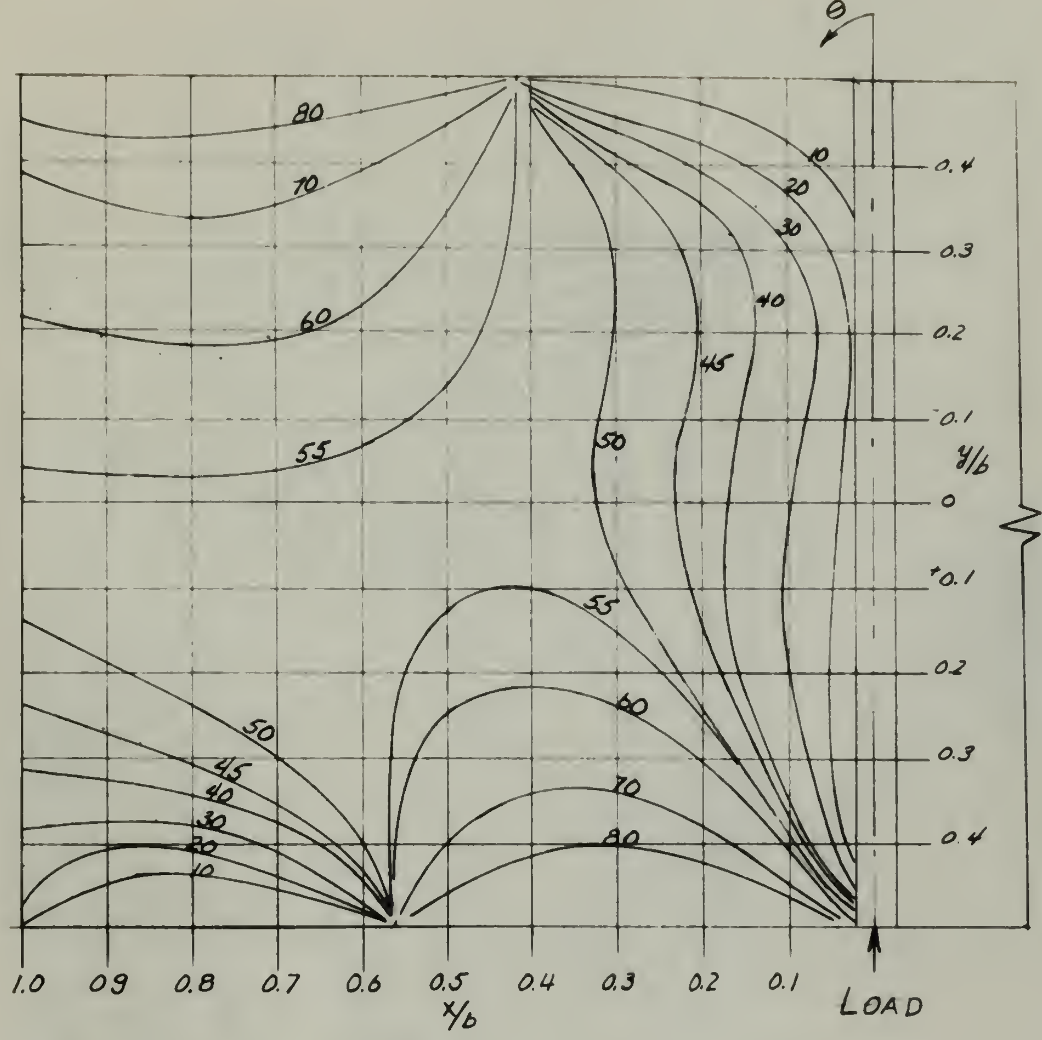


Figure VI (b)
ISOCLINIC PATTERN
Aspect Ratio 2:1
Stiffened Plate

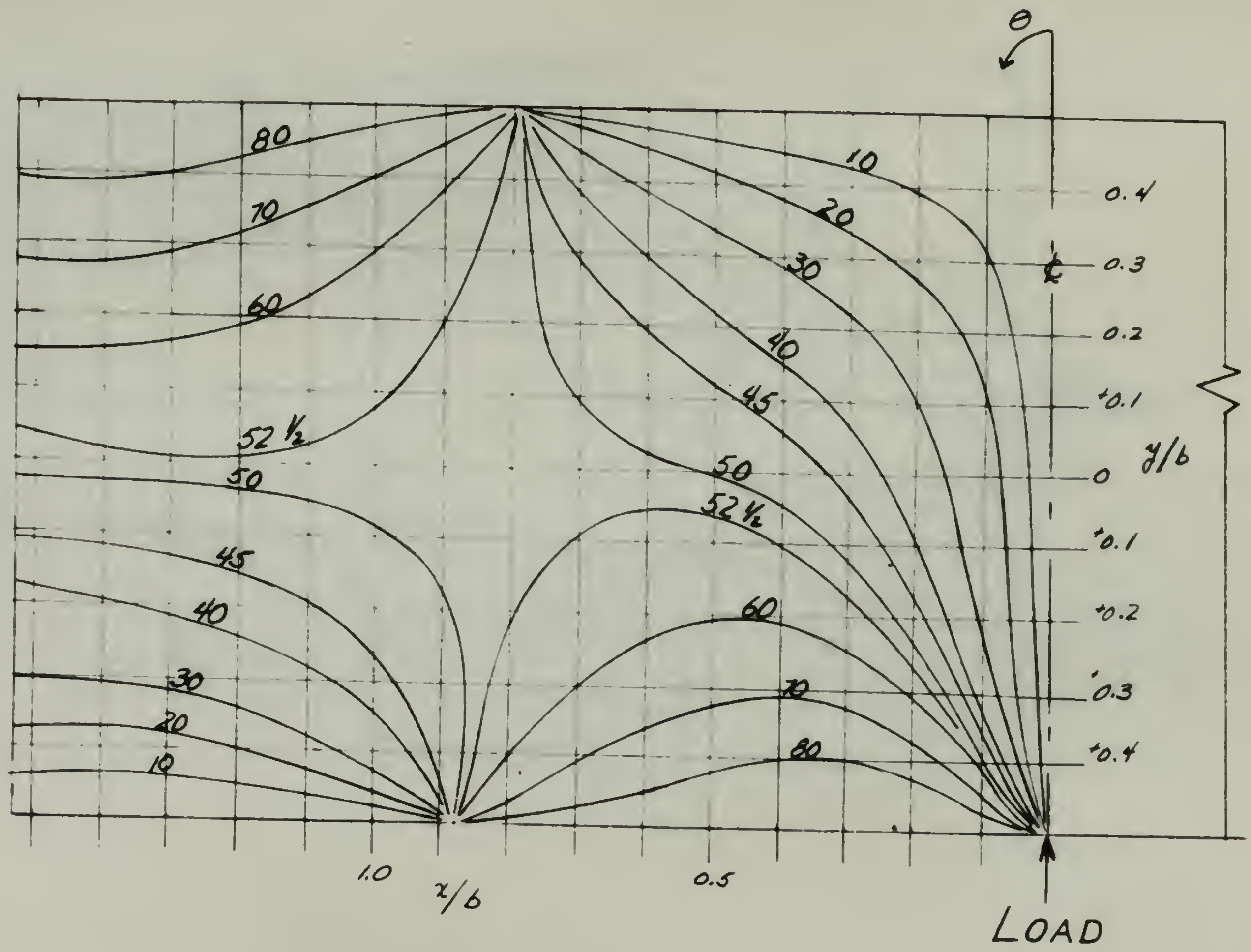


Figure VII (a)
ISOCLINIC PATTERN
Aspect Ratio 3:1
Unstiffened Plate

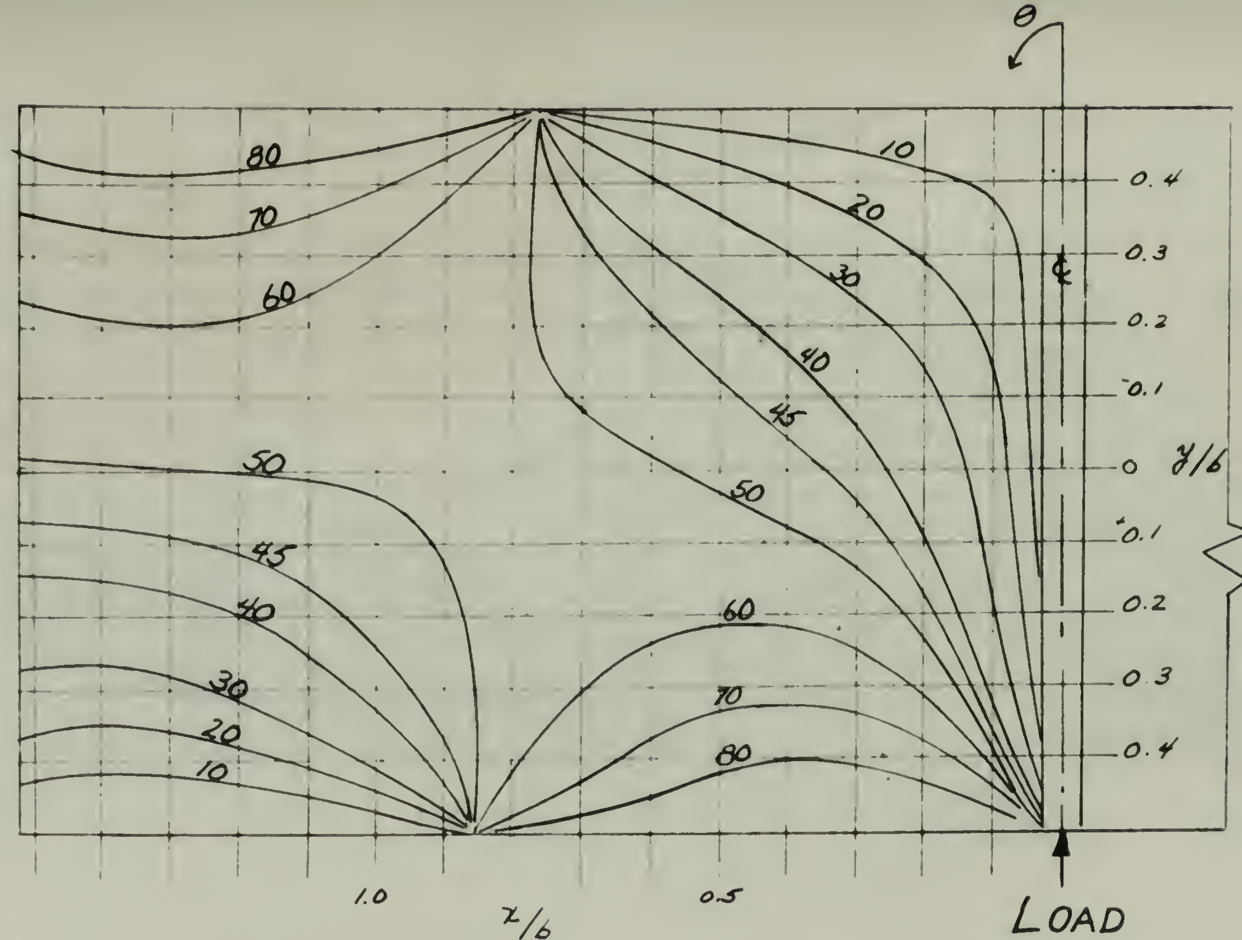


Figure VII (b)
ISOCLINIC PATTERN
Aspect Ratio 3:1
Stiffened Plate

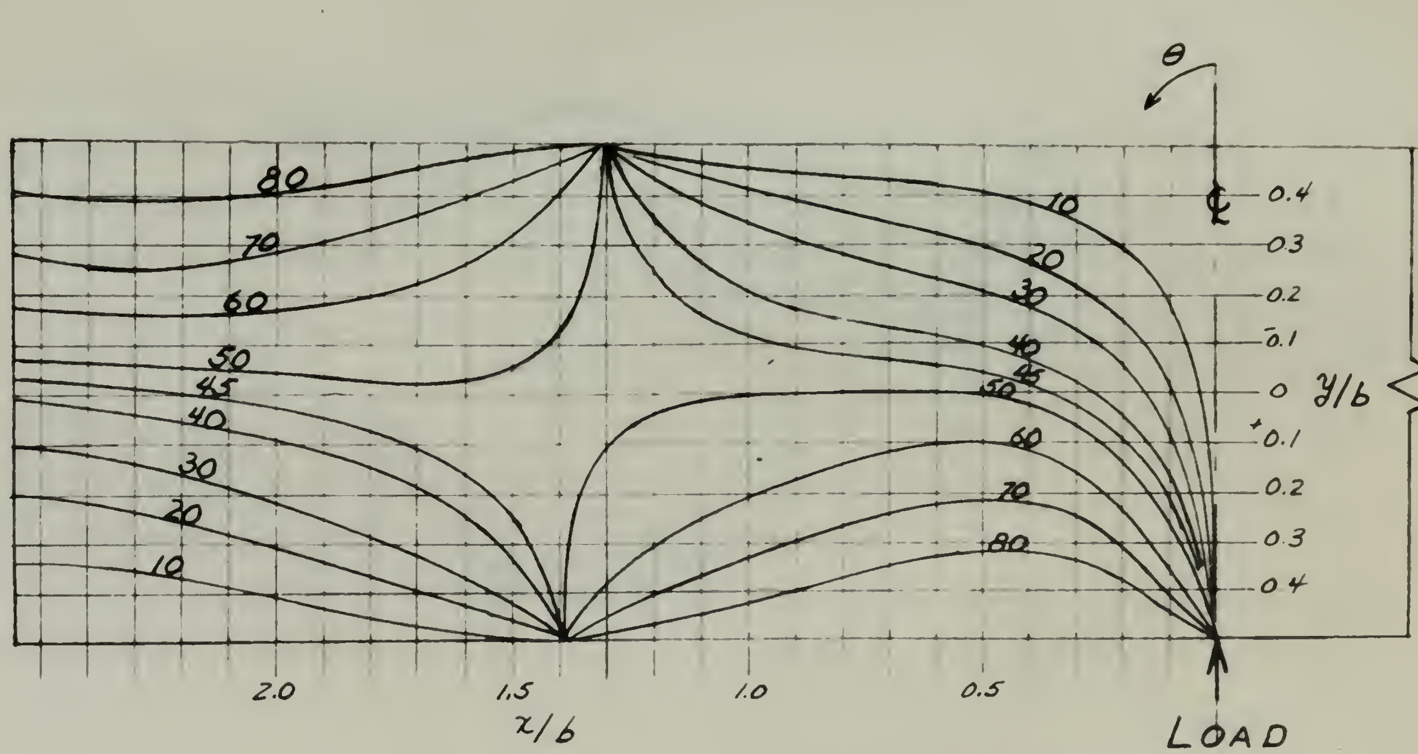


Figure VIII (a)
ISOCLINIC PATTERN
Aspect Ratio 5:1
Unstiffened Plate

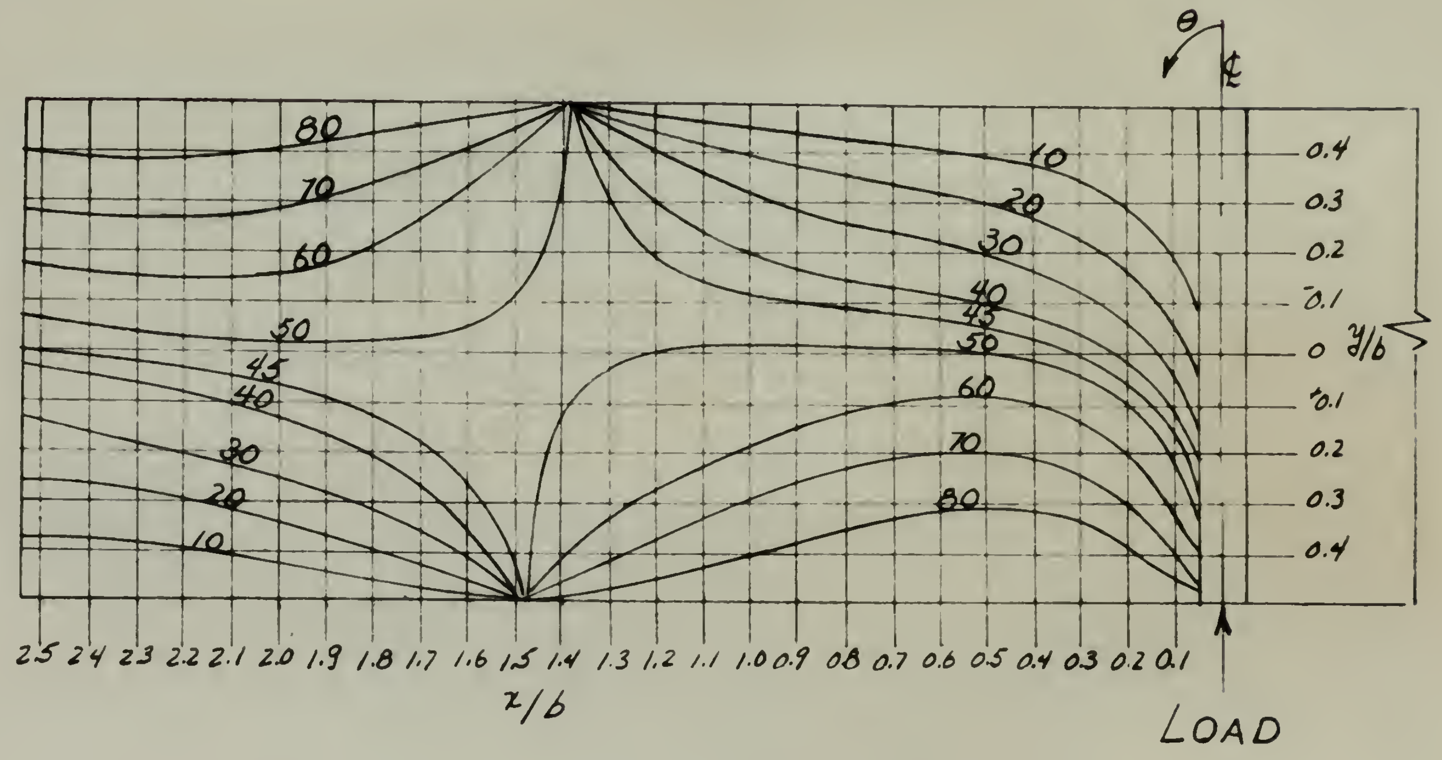


Figure VIII (b)
ISOCLINIC PATTERN
Aspect Ratio 5:1
Stiffened Plate

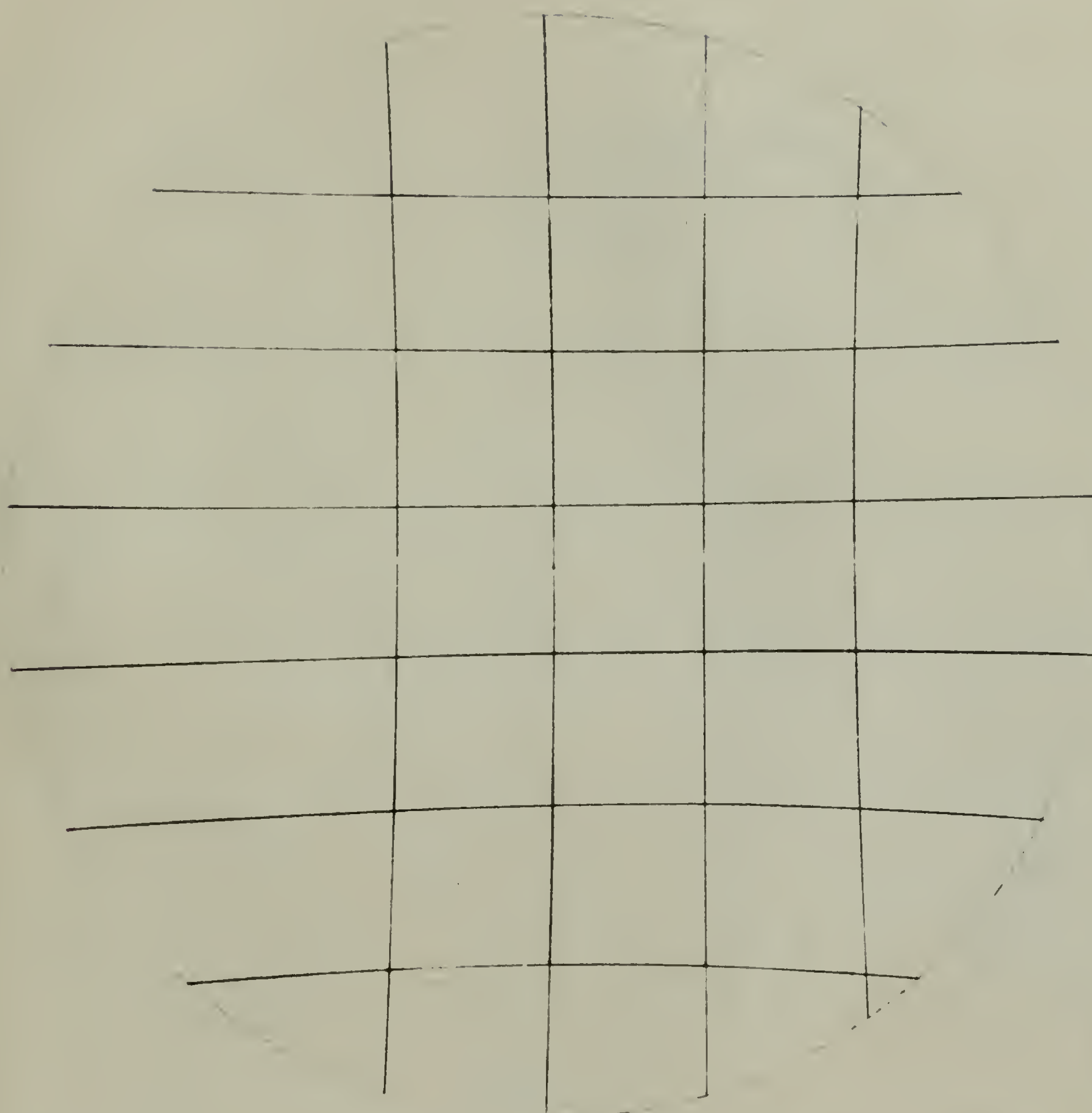


Figure IX

DISTORTION PATTERN
of
One-inch Square Grid

P (tensile) stress

Q (compressive) stress

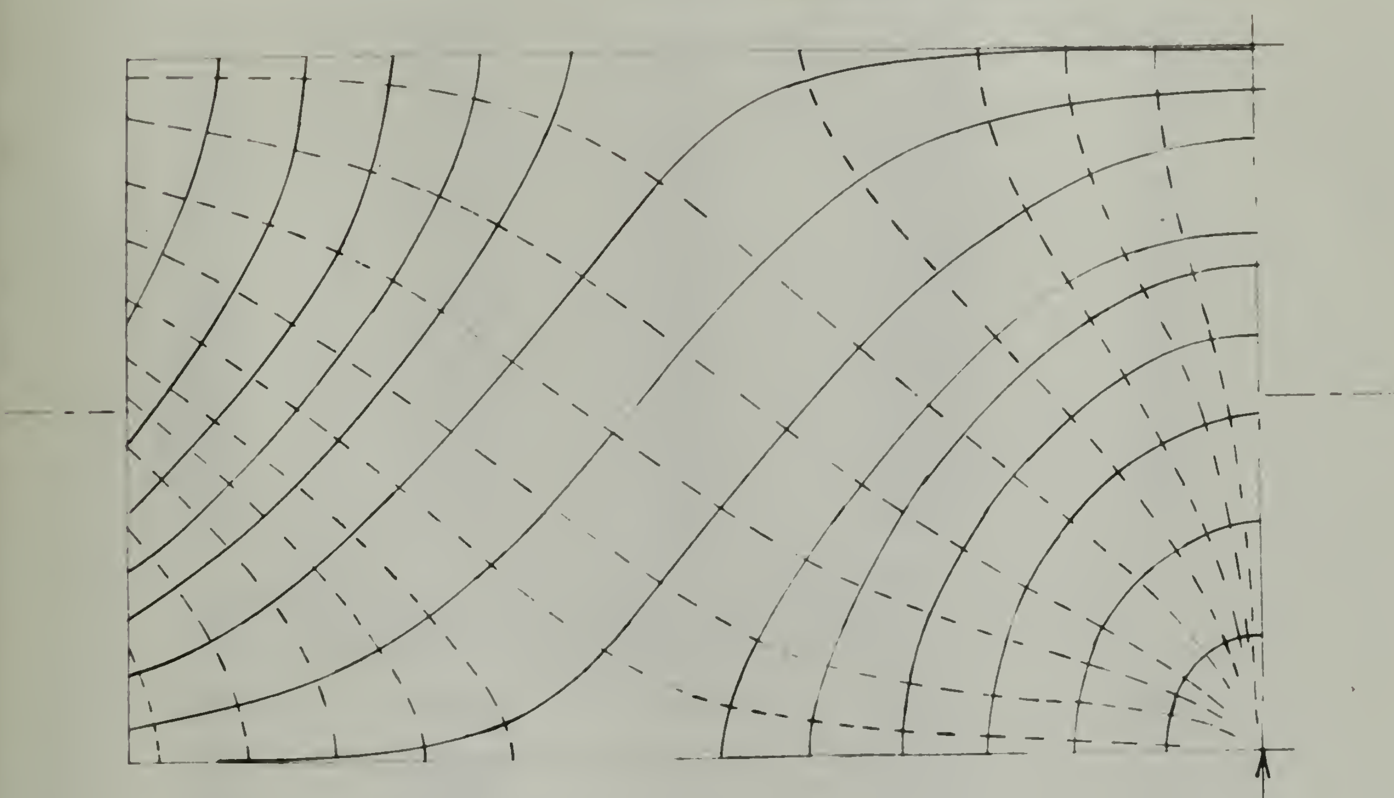


Figure X

STRESS TRAJECTORIES
Aspect Ratio 3:1
Unstiffened Plate

TABLE I

Calibration of Load Pointer

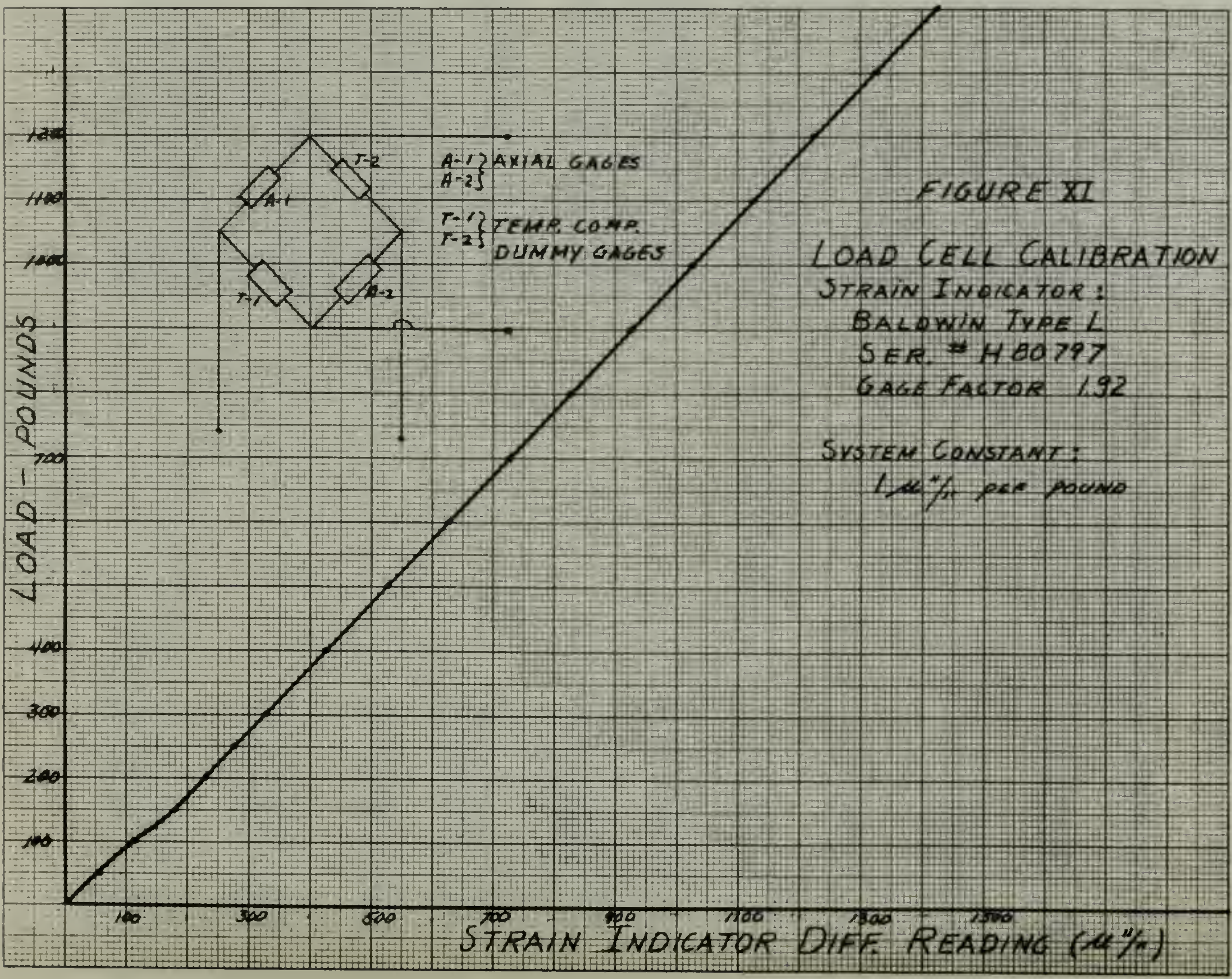
Gage: A-2; axial gage; Type A-7; G.F. 1.95				
Load	Reading	Diff.	Reading	Diff.
0	6-0095	0	6-0085	0
100	6-0040	55	6-0038	47
200	4-1984	111	4-1980	105
300	4-1935	160	4-1930	155
400	4-1890	205	4-1880	205
500	4-1835	260	4-1840	245
600	4-1783	312	4-1790	275
700	4-1735	360	4-1745	340
800	4-1690	405	4-1695	390
900	4-1640	455	4-1650	435
1000	4-1590	505	4-1600	485
1100	4-1540	555	4-1550	535
1200	4-1495	600	4-1500	585
1300	4-1445	650	4-1452	632
1400	4-1395	700	4-1410	675
1500	4-1350	745	4-1360	725

Gage: A-1; axial gage; Type A-7; G.F. 1.95				
Load	Reading	Diff.	Reading	Diff.
0	6-0310	0	6-0305	0
100	6-0260	50	6-0250	55
200	6-0220	90	6-0200	105
300	6-0170	140	6-0150	155
400	6-0125	185	6-0100	205
500	6-0080	230	6-0040	265
600	6-0030	280	4-1990	315
700	4-1980	330	4-1940	365
800	4-1940	370	4-1890	415
900	4-1890	420	4-1835	475
1000	4-1850	460	4-1780	525
1100	4-1805	505	4-1730	575
1200	4-1760	550	4-1680	625
1300	4-1715	605	4-1630	675
1400	4-1665	645	4-1580	725
1500	4-1622	648	4-1530	775

TABLE I (Cont.)

Gage: A-1 & A-2 in opposite arms of bridge; 2 dummy gages
Gage Factor set at 1.92

Load	Reading	Diff.
0	12-1188	0
50	12-0132	56
100	12-0075	113
150	12-0008	180
200	10-1960	228
250	10-1910	278
300	10-1860	328
350	10-1810	378
400	10-1760	428
450	10-1710	478
500	10-1660	528
600	10-1560	628
700	10-1460	728
800	10-1360	828
900	10-1260	928
1000	10-1160	1028
1100	10-1060	1128
1200	10-0955	1223
1300	10-0860	1328
1400	10-0765	1423
1500	10-0660	1528


LOAD - POUNDS
STRAIN INDICATOR DIFF. READING ($\mu''/_{''}$)
A-1
A-2
T-1
T-2
A-1 } AXIAL GAGES
A-2 }
T-1 } TEMP. COMP.
T-2 } DUMMY GAGES
FIGURE XI
LOAD CELL CALIBRATION
STRAIN INDICATOR:
BALDWIN TYPE L
SER. # H 80797
GAGE FACTOR 1.92
SYSTEM CONSTANT:
$1 \mu''/_{''}$ per pound

E. LITERATURE CITATIONS

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